Unified Engineering Problem Set Lectures: M1, M2, M3, M4*(look-ahead)*
Week 2 Spring, 2008 Units: M4.1, M4.2, M4.3*(look-ahead)*

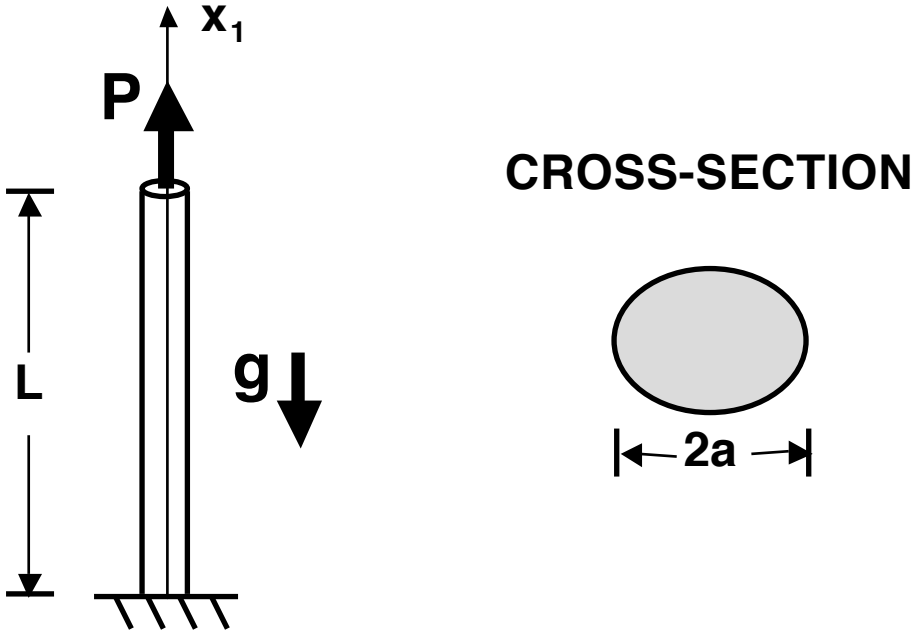
M2.1 ***(5 points)*** This problem serves as a mechanism to review the key concepts and governing equations in general elasticity.

(a) Write out fully, in tensor notation, the governing *independent* equations of elasticity for a solid body. Identify the key assumptions associated with each set of these equations and the underlying fundamental(s) upon which these are based.

(b) There are also several other equations, known as "compatibility equations". Describe what they are and from where they come. Why aren't these also independent equations?

(c) If engineering notation is used, do these equations change? How? Indicate this either through careful and complete description or by writing out the equations that change. In either case, highlight any key differences.

M2.2 ***(15 points)*** An elliptical rod is attached to the ground as indicated in the accompanying figure. The rod is of length L, major diameter, 2a, and the cross-section has an aspect ratio of 3/4. The rod is made of an isotropic material with a longitudinal modulus of E, Poisson's ratio of ν, shear modulus of G, and density of ρ. The rod is subjected to a tensile load at its tip given as of magnitude P. The entire arrangement is subjected to a gravity field of value g.



In the problem, include the effects of the mass of the rod. However, it is suggested to consider the effects of the tip load and the effects of the rod mass separately for parts (a) through (c).

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

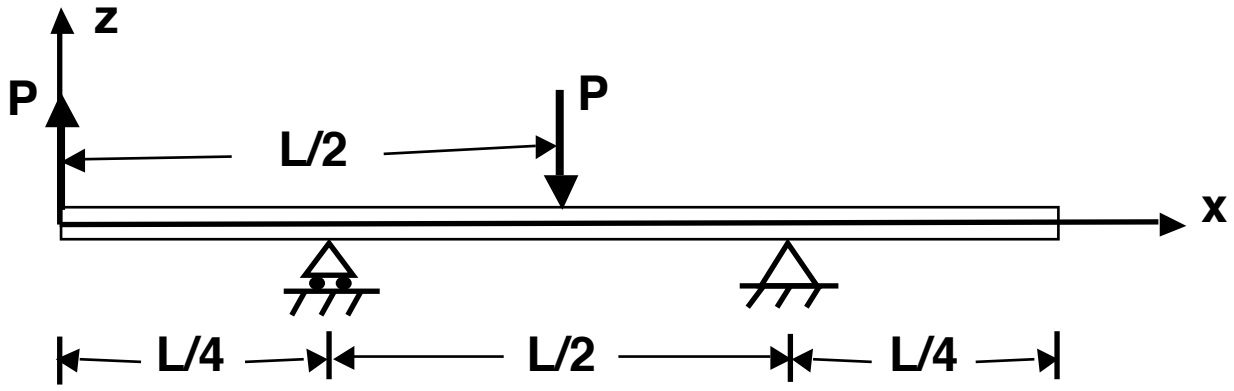
(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

(e) If the cross-section of the rod varies with the aspect ratio changing such that the major diameter has a value of 2a at the top of the rod and a value of 3/2 a (circular cross-section) at the bottom of the rod, can the rod model still be used? Why or why not? Be sure to explain clearly using equations if/as needed.

(f) The final objective is to design this rod as a structural fuse with failure occurring at the attachment to the ground.. Knowing that failure occurs when the stress in the rod reaches a material ultimate value of σ_{ult}, describe how this can be done. Use equations and solve for pertinent values as appropriate.

M2.3 ***(10 points)*** A beam of total length L has a roller support at the quarter point (L/4) and is pinned at the three-quarters point (3/4 L). The beam is loaded by a concentrated upward load of magnitude P at the beam tip (x = 0) and a concentrated downward load of magnitude P at the half point (x = L/2).



(a) Determine the reactions for this structural configuration.

(b) Determine the shear and moment diagrams.